

# Kinematics of Internal Space associated with the TEGR

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## Abstract

In the context of Teleparallel Equivalent of General Relativity - TEGR - we have obtained, through the second kind gauge transformations, the most fundamental transformations, namely, the first kind ones. We show that considering the possibility of decomposing the components of the tetrad field as a trivial part plus some potential besides the usual translational and Lorentz potentials, there is also the possibility that the symmetry group of internal space be a generalization of the Poincaré group. Still in the analysis of transformations in the internal space, we saw that for the case in which the decomposition of the tetrad field components includes only a trivial part plus a translational potential, we recover just the translational group. Moreover, the internal space of gravity must be the very physical space - on a local scale - in such a way that the gauge symmetry group is the kinematical group of the spacetime itself. This group was obtained, and seems to generalize the Poincaré group.

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# 1 Introduction

Gauge theories of gravitation are an attempt to describe such interaction using techniques well established in the description of the other fundamental interactions. Moreover, the study of symmetries and conservation laws in physical space is a powerful tool for understanding the fundamentals of mechanics. The idea of to extrapolate and to use the same kind of apparatus in an internal space opens a window of possibilities; this was exactly what allowed us to understand the real essence of the "charge": quantities conserved under the action of a particular group of symmetry in internal space [1].

The existence of an internal space, in turn, is associated with extra degrees of freedom which are evident in the hamiltonian formulation of the theory. These theories are said to be constrained, since its phase space has decreased [2]. Although in the study of constrained hamiltonian systems the appearance of extra degrees of freedom (internal space) is a consequence of the size of the configuration space, without any physical notion of real essence of this space, we know they are degrees of freedom associated with the source field; a consequence of the invariance of the total Lagrangian under transformations in this field.

The equivalence principle states that the equations of special relativity must be recovered in a locally inertial coordinate system in which gravitational effects are absent. Thus, based on this principle, it would be natural to expect that gravitation had a local Poincaré symmetry, and it was possible to describe it as a genuine gauge theory for this group. In fact, it is possible. [3]. However there are evidences, both theoretical and experimental/observational, that from the most fundamental point of view (beyond the standard model) the Lorentz symmetry is broken [4, 5, 6, 7, 8, 9, 10]. This would eliminate the Poincaré group as local symmetry group of gravity, leaving room only for the translational sector or other more general.

The TEGR is a particular case of a more general theory that has a set of three free parameters, the Teleparallel Gravity, which is fully equivalent to General Relativity [11, 12, 13, 14, 15]. To better understand its internal structure, one should apply the tools mentioned in the paragraphs above. As will be seen, we show that starting from the TEGR Lagrangian presented in the literature, we conclude that the internal space associated with the TEGR seems to be invariant under the action of a group which generalizes the Poincaré group.

Notation: the Greek alphabet is used to represent space-time indices ( $\alpha, \beta, \chi, \dots = 0, 1, 2, 3$ ), the first half of the Latin alphabet ( $a, b, c, \dots = 0, 1, 2, 3$ ) represents indices of the internal space, or gauge, and the second half of the latin alphabet ( $i, j, k, \dots$ ) assumes the values of 1, 2 and 3 of the space-time.

## 2 The TEGR and their First Class Constraints as Generators of the Gauge Transformations

The Lagrangian density associated with TEGR has the form <sup>1</sup> [13]:

$$\mathcal{L} = \frac{h}{2k^2} \left[ \frac{1}{4} T^\rho{}_{\mu\nu} T^\mu{}_{\rho}{}^{\nu} + \frac{1}{2} T^\rho{}_{\mu\nu} T^{\nu\mu}{}_{\rho} - T_{\rho\mu}{}^{\rho} T^{\nu\mu}{}_{\nu} \right] \quad (1)$$

with  $h = \det(h^a{}_\mu)$  and  $k = \frac{8\pi G}{c^4}$ . This expression can be rewritten in a more elegant form to get [16]:

$$\mathcal{L} = \frac{h}{4k^2} S^{\rho\mu\nu} T_{\rho\mu\nu} , \quad (2)$$

where

$$S^{\rho\mu\nu} = -S^{\rho\nu\mu} \equiv \frac{1}{2} [K^{\mu\nu\rho} - g^{\rho\nu} T^{\theta\mu}{}_{\theta} + g^{\rho\mu} T^{\theta\nu}{}_{\theta}] \quad (3)$$

and  $K^{\mu\nu\rho}$  is the contortion tensor given by

$$K^{\mu\nu\rho} = \frac{1}{2} T^{\nu\mu\rho} + \frac{1}{2} T^{\rho\mu\nu} - \frac{1}{2} T^{\mu\nu\rho} . \quad (4)$$

The procedure of a Legendre transformation from the previous Lagrangian density is not sufficient to get its Hamiltonian version, since it is not possible to isolate all the terms of velocities depending on the momenta [17]. To circumvent this problem, we should simplify this Lagrangian density, making it linear before proceeding [18]; this "linearization" is done by introducing auxiliary fields  $\phi_{abc} = -\phi_{acb}$  that will be related to the torsion tensor. The first order differential Lagrangian formulation in empty space-time reads

$$\mathcal{L} = kh\Lambda^{abc} (\phi_{abc} - 2T_{abc}) , \quad (5)$$

where  $T_{abc} = h_b{}^\mu h_c{}^\nu T_{a\mu\nu}$ ,  $\Lambda^{abc}$  is defined by

$$\Lambda^{abc} = \frac{1}{4} (\phi^{abc} + \phi^{bac} - \phi^{cab}) + \frac{1}{2} (\eta^{ac} \phi^b - \eta^{ab} \phi^c) , \quad (6)$$

and  $\phi_b = \phi^a{}_{ab}$ . After a long development, the first class secondary constraints are found <sup>2</sup>

$$\chi_c = h_c{}^0 \mathcal{H}_0 + h_c{}^i F_i , \quad (7)$$

where

$$\begin{aligned} \mathcal{H}_0 &= -h_{a0} \partial_k \Pi^{ak} - \frac{kh}{4g^{00}} (g_{ik} g_{jl} P^{ij} P^{kl} - \frac{1}{2} P^2) \\ &+ kh \left( \frac{1}{4} g^{im} g^{nj} T^a{}_{mn} T_{aij} + \frac{1}{2} g^{nj} T^i{}_{mn} T^m{}_{ij} - g^{ik} T^j{}_{ji} T^n{}_{nk} \right) \end{aligned} \quad (8)$$

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<sup>1</sup>The torsion is defined by  $T^\rho{}_{\mu\nu} \equiv \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu}$ . The object  $\Gamma^\rho{}_{\nu\mu}$  is the Weitzenbock connection defined by  $\Gamma^\rho{}_{\nu\mu} \equiv h_a{}^\rho \partial_\mu h^a{}_\nu$ .

<sup>2</sup>These are the relevant constraints in the calculation of the gauge transformations.

and

$$F_i = h_{ai} \partial_k \Pi^{ak} - \Pi^{ak} T_{aki} + \Gamma^m T_{0mi} + \Gamma^{lm} T_{lmi} + \frac{1}{2g^{00}} (g_{ik} g_{jl} P^{kl} - \frac{1}{2} P) \Gamma^j. \quad (9)$$

Furthermore, the objects were defined:

$$\Pi^{ai} = -4kh\Lambda^{a0i}, \quad (10)$$

$$\begin{aligned} P^{ik} &= \frac{1}{2kh} (h_c^i \Pi^{ck} + h_c^k \Pi^{ci}) + g^{0m} (g^{kj} T_{mj}^i + g^{ij} T_{mj}^k - 2g^{ik} T_{mj}^j) \\ &+ (g^{km} g^{0i} + g^{im} g^{0k}) T_{mj}^j, \end{aligned} \quad (11)$$

$$\Gamma^{ik} = \frac{1}{2} (h_c^i \Pi^{ck} - h_c^k \Pi^{ci}) - kh [-g^{im} g^{kj} T_{mj}^0 + (g^{im} g^{0k} - g^{km} g^{0i}) T_{mj}^j] \quad (12)$$

and

$$\Gamma^k = \Pi^{0k} + 2kh (g^{kj} g^{0i} T_{ij}^0 - g^{0k} g^{0i} T_{ij}^j + g^{00} g^{ik} T_{ij}^j); \quad (13)$$

where  $\Pi^{ai}$  are the momenta canonically conjugated to  $h_{ai}$ . Being the constraints first class we can use then to calculate the transformations generated by the constraints, which do not alter the physical state of the system (gauge transformations) [2]:

$$\begin{aligned} \delta h^b_\rho(x) &= \int d^3x' [\varepsilon_1^a(x') \{h^b_\rho(x), \Phi_a(x)\} + \varepsilon_2^a(x') \{h^b_\rho(x), \chi_a(x)\}] \\ &= \int d^3x' \varepsilon_1^a(x') \left( \frac{\delta h^b_\rho(x)}{\delta h^c_\beta(x')} \frac{\delta \Phi_a(x)}{\delta \Pi_c^\beta(x')} - \frac{\delta h^b_\rho(x)}{\delta \Pi_c^\beta(x')} \frac{\delta \Phi_a(x)}{\delta h^c_\beta(x')} \right) \\ &+ \int d^3x' \varepsilon_2^a(x') \left( \frac{\delta h^b_\rho(x)}{\delta h^c_\beta(x')} \frac{\delta \chi_a(x)}{\delta \Pi_c^\beta(x')} - \frac{\delta h^b_\rho(x)}{\delta \Pi_c^\beta(x')} \frac{\delta \chi_a(x)}{\delta h^c_\beta(x')} \right), \end{aligned} \quad (14)$$

that results in [17]

$$\delta h^b_\rho = \delta_\rho^0 \varepsilon_1^b + \nabla_\rho \varepsilon_2^b, \quad (15)$$

with

$$\nabla_\rho \varepsilon_2^b \equiv \delta_\rho^i \partial_i \varepsilon_2^b + \omega^b_{a\rho} \varepsilon_2^a \quad (16)$$

and

$$\begin{aligned} \omega^b_{a\rho} &\equiv -\frac{1}{g^{00}} \delta_\rho^i h_a^0 h^b_i g^{0\mu} T_{j\mu}^j - \frac{1}{2g^{00}} \delta_\rho^i h_a^0 h^{b0} T_{0i}^0 + \frac{3}{2g^{00}} \delta_\rho^i g^{0b} h_{ai} g^{0\mu} T_{j\mu}^j \\ &+ \frac{1}{2g^{00}} \delta_\rho^i h^{b0} h_{ai} g^{0\mu} T_{j\mu}^j + \frac{1}{2g^{00}} \delta_\rho^i h_a^0 g^{0b} T_{0i}^0 + \frac{3}{2} \delta_\rho^i h^b_\mu h_a^\nu T^{\mu}_{i\nu} \\ &+ \delta_\rho^i g^{0b} g_{0\mu} h_a^\nu T^{\mu}_{i\nu} - \frac{1}{2} \delta_\rho^i g_{i\mu} h^{b\nu} h_a^\alpha T^{\mu}_{\nu\alpha} + \frac{1}{2} \delta_\rho^i h_{ai} h^{b\mu} T^0_{0\mu} \\ &- \delta_\rho^i h^b_i h_a^\mu T^0_{0\mu} + \frac{1}{2} \delta_\rho^i \delta_a^b T^0_{0i} \end{aligned} \quad (17)$$

playing the role of covariant derivative and connection, respectively. We can go ahead and rewrite (15) as follows:

$$\delta h^b_\rho = \delta_\rho^0 \varepsilon_1^b + \delta_\rho^i \partial_i \varepsilon_2^b + \omega^b_{a\rho} \varepsilon_2^a. \quad (18)$$

Introducing now the following relation between the parameters  $\varepsilon_1^b$  and  $\varepsilon_2^b$

$$\varepsilon_1^b = \partial_0 \varepsilon_2^b, \quad (19)$$

we have:

$$\delta h^b{}_\rho = \partial_\rho \varepsilon_2^b + \omega^b{}_{a\rho} \varepsilon_2^a. \quad (20)$$

The subindex 2 can now be ignored,

$$\delta h^b{}_\rho = \partial_\rho \varepsilon^b + \omega^b{}_{a\rho} \varepsilon^a \equiv \nabla'_\rho \varepsilon^b. \quad (21)$$

The above transformations allow a direct analogy with the gauge transformations obtained in the Yang-Mills theory.

### 3 Kinematics of Internal Space

The transformations (21) enable a fundamental analysis to understand the kinematics of the internal space (gauge symmetry group). We have:

$$\delta h^b{}_\rho = \partial_\rho \varepsilon^b + \omega^b{}_{a\rho} \varepsilon^a. \quad (22)$$

and motivated by the soldering property [19], we can decompose  $h^b{}_\rho$  as follows

$$h^b{}_\rho = \delta_\rho^b + A^b{}_\rho + A_a{}^b{}_\rho x^a + \dots \quad (23)$$

Even if there is no gauge potential (gravity), the trivial part  $\delta_\rho^b$  remains "linking" the physical and internal spaces. Although we have no idea of what would represent other potentials, besides the already well known  $A^b{}_\rho$  and  $A_a{}^b{}_\rho$ <sup>3</sup>, we have kept open the possibility that they exist. Armed with this decomposition, it is possible to write

$$\delta h^b{}_\rho = h^{b'}{}_\rho - h^b{}_\rho = \delta_\rho^{b'} - \delta_\rho^b + \delta(A^b{}_\rho) + \delta(A_a{}^b{}_\rho x^a) + \dots, \quad (24)$$

or, using (22)

$$\frac{\partial}{\partial x^\rho} (x^{b'} - x^b) = \partial_\rho \varepsilon^b + \omega^b{}_{a\rho} \varepsilon^a - \delta A^b{}_\rho - \delta(A_a{}^b{}_\rho x^a) - \dots \quad (25)$$

From this point it is possible, by integrating in  $x^\rho$ , to get the following expression for the internal space transformations:

$$\delta x^b = \epsilon^b + C^b - F^b - \varpi^b{}_a x^a - \dots \quad (26)$$

with

$$\begin{aligned} \epsilon^b &\equiv \int \partial_\rho (\varepsilon^b) dx^\rho, \\ C^b &\equiv \int \omega^b{}_{a\rho} \varepsilon^a dx^\rho, \\ F^b &\equiv \int \delta A^b{}_\rho dx^\rho, \\ \varpi^b{}_a x^a &\equiv \int \delta(A_a{}^b{}_\rho x^a) dx^\rho. \end{aligned} \quad (27)$$

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<sup>3</sup>Components of the gauge potential associated with the translational and Lorentz sectors, which arises as compensating fields that guarantee the invariance of Lagrangian under first kind gauge transformations.

Or, rearranging terms:

$$\delta x^b = \epsilon'^b + \varpi_a^b x^a + \dots \quad (28)$$

here  $\epsilon'^b \equiv \epsilon^b + C^b - F^b$ , and the negative sign of  $\varpi_a^b$  was absorbed by  $\varpi_a^b$ . In this case, assuming that the last integral in (27) results in the usual Lorentz transformations, we clearly have a generalization of the Poincaré transformations

$$\delta x^b = \epsilon^b + \varpi_a^b x^a; \quad (29)$$

however, this generalization only exists for cases in which we include potentials beyond the usual  $A_\rho^b$  and  $A_\rho^{ab}$ <sup>4</sup>, which would imply the existence of other degrees of freedom associated with these potentials.

From the expressions (27) and (28) it can be noted that for the case in which  $h_\rho^b$  is decomposed only as  $\delta_\rho^b + A_\rho^b$ , all the "rotation" matrices  $\varpi_a^b$  are zero, hence:

$$\delta x^b = \epsilon^b, \quad (30)$$

that is exactly a translation in the internal space. This is in total agreement with the literature of gauge theories to the translation group [12], in which the fundamental object is the potential  $A_\rho^b$ .

Regardless of what values are calculated in the integrations (27), it seems reasonable to think that their results will take the form (28), since through it we can get both Poincaré as translations, depending on how we make the decomposition of the tetrad field components  $h_\rho^b$ .

## 4 Universality via gauge approach

From the viewpoint of gauge theories, the gauge fields "appear" as compensating fields, that guarantee the invariance of the Lagrangian under the action of a transformation group acting on the source field (first kind gauge transformations) [1]. Thinking this way, the analysis of a Lagrangian without sources - anywhere - is merely an academic exercise, don't having much physical support; the only way to "generate" the gauge field is through the action of the symmetry group of the internal space (variation of the source field). For the case of gravitation, however, the previous statement is not necessarily true, since the free Lagrangian leads to the field equations that contain a term of "vacuum source"<sup>5</sup>: in other words, gravitation is nonlinear. When trying to do a mental exercise and view the generation of gauge fields in the case of gravitation, we face the following problem. Let us consider an arbitrary source of gravity defined in an internal space. Acting with the symmetry group of this space it emerges a distortion in a compensatory way (tension, making an analogy with a fabric) in physical space, which manifests itself through the gravitational field. This gravitational field, in turn, having a no null energy should also generate an additional distortion in space (second order effect), but, being defined in physical space, how could it restart the cycle? It is necessary that the gravitational field interferes locally in the

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<sup>4</sup>In the case that the last integral in (27) does not results in the Lorentz transformations, we have a generalization although we have not included other potentials.

<sup>5</sup>Gravitational Maxwell's equations enables a proper analysis of this issue [20].

internal space for it generates, in turn, a new perturbation in the physical space (second order field), and this process repeats indefinitely.

Observing the first and second kind gauge transformations it is possible to infer how the process occurs. We have, after a gauge transformation in the internal space,

$$x^b \rightarrow x^{b'} = x^b + \delta x^b = x^b + \epsilon'^b + \varpi'^b{}_a x^a + \dots, \quad (31)$$

and consequently the physical space responds with the emergence of  $h^b{}_\rho$ . This, in turn, varies with  $\varepsilon^b$  according to

$$\delta h^b{}_\rho = \nabla'_\rho \varepsilon^b = \partial_\rho \varepsilon^b + \omega^b{}_{a\rho} \varepsilon^a. \quad (32)$$

The connection  $\omega^b{}_{a\rho}$  can be divided in two parts; the first one leads algebra indices into algebra indices again, as in the Yang-Mills theory. The second, in turn, leads algebra indices into physical space indices:

$$\omega^b{}_{a\rho} \varepsilon^a = (\dots) \delta^b_a \varepsilon^a + [(\dots) h_a{}^0 + (\dots) h_{ai} + (\dots) h_a{}^\nu] \varepsilon^a, \quad (33)$$

or still

$$\omega^b{}_{a\rho} \varepsilon^a = (\dots) \varepsilon^b + [(\dots) \varepsilon^0 + (\dots) \varepsilon_i + (\dots) \varepsilon^\nu]. \quad (34)$$

The first part is analogous to the Yang-Mills connection. The second part, however, allows that  $\varepsilon$ , now with spacetime index, to calibrate objects defined in this space. Note that this is not only a consequence of the locality of gauge transformations; the effect of the first kind transformations is just "to force" the emergence of the field. However, it does not ensure that this field generates new transformations; these arise as consequence of the parameters  $\epsilon'^b$  have the possibility of be written as being  $\epsilon'^\mu$ . In addition, the form of transformations (31) is not fixed, it varies with  $x^\mu$ ; the first two integrals in (27) depend on  $\varepsilon$ , and therefore they have different results for each of the possible indices for this parameter, internal or physical.

Another key issue is that for the case of gauge theories such as electromagnetism, for example, the first kind transformations are represented by the action of the symmetry group in a given field source. For the case of gravitation, however, the group's action takes place directly on the base that defines the internal space  $x^b$ , not on a source field defined in this space<sup>6</sup>. This allows us to conjecture that the source of gravity must be associated to the existence of their own internal space, and not the sources defined therein, as with other interactions.

The equivalence principle, or equivalently the universality of free fall, leaves a tiny gap to the issue of symmetry of internal space. In fact, being the gravitation universal, its symmetry group of the internal space should act on any field that has non-zero energy, thus ensuring their coupling to this field. However, the very usual concept of gauge symmetry rejects this idea, i.e. the gauge group acts only on fields with specific properties<sup>7</sup> and therefore defined in a special space, called internal space. One way to around this issue is to assume that the internal space of gravity is the very physical space in a "local scale", in such way that thinking in a symmetry group for this space is now the same as thinking in a kinematics

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<sup>6</sup>It is clear that for cases in which there is a source field, besides the gravitational field, the group will also act in this field.

<sup>7</sup>Such as the gauge currents  $J_a{}^\mu$ .

to the physical space. This kinematics is given by the set of transformations (28), which can generalize the usual transformations of special relativity. For obvious reasons, this alternative is valid only if the gravitation is in fact universal. Note that the scenario above is quite different from a mere extrapolation of gravity for the context of gauge theories.

## 5 Final remarks

Through the gauge transformations of the second kind, it was possible to get the most fundamental transformations, i.e., the first kind. We show that with the possibility of decomposing the tetrad field components as a trivial part plus some potential besides the usual Lorentz and translational potentials, there is still the possibility that the symmetry group of the internal space be a generalization of the Poincaré group. Still in the analysis of transformations in the internal space, we saw that for the case in which the decomposition of the tetrad field components mentioned above includes only a trivial part plus the translational potential, we recover the group of translations. This is in complete agreement with the literature.

The *spin* connection present in (17), and rewritten in (33) and (34), allows that internal space indices be taken to physical space indices, i.e., the parameters of gauge transformation can also be defined on the base manifold. This unique peculiarity of gravitation makes impossible that the parameters defined in (27) be interpreted as phases of wave functions defined in internal space, being this peculiarity related to universality of gravitation, i.e., the universality allows that objects be defined both in physical space or in the internal space. In short, given a source field  $\psi$ <sup>8</sup>,  $h^a_\mu$  can be any  $h^a_\mu + \delta h^a_\mu$ , with  $\delta h^a_\mu$  given by (22). When choosing a specific calibration, through the introduction of new second class constraints,  $\psi$  and  $h^a_\mu$  are also specified, i.e. all degrees of freedom have now been transferred to the physical space. This specification, however, was not made at the expense of choosing a particular phase of a wave function defined in internal space, but choosing parameters, even without a precise physical interpretation, which can be defined in both spaces. Is also worth mentioning that being gravitation universal, its internal space must be the actual physical space (local scale), and therefore its symmetry group of internal space is the kinematical group of physical space itself. This group was obtained, and is represented by the set of transformations (28).

Finally, we are investigating the possible relationship between the clear non-compact appearance of the symmetry group of the internal space<sup>9</sup> and the existence of gravitational singularities. The "non-compactness" allows the internal space be "bigger" than the very physical space (in a local scale), thus, by making a first kind gauge transformation it is possible that the physical space, being locally attached to the internal space, has its structure "stretched" beyond of supportable. As a result, the fabric of the physical space would be "ripped" thus creating a singularity. A complete analysis of this issue will be made elsewhere.

## Acknowledgements

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<sup>8</sup>From the nonlinearity of field equations, we know that  $\psi$  can be associated with the gravitational field itself. However, as already said, it does not mean that there are other sources.

<sup>9</sup>This feature can be seen in (28).



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